

Lecture 13 – Stereopsis

(Schwartz Chapter 10; Steinman Chapter 7)

OVERREVIEW OF DEPTH PERCEPTION

Visual perception

A. What things are (image processing)

1. Spatial vision (formation of the retinal image, neural processing, contrast sensitivity, etc.)
2. Color perception
3. Temporal vision
4. Visual adaptation

B. Where they are (space perception)

1. Visual direction
 - * Oculocentric localization
 - * Egocentric localization
2. Depth perception
 - * Monocular depth cues
 - Pictorial cues (size, interposition, linear perspective, shadows, etc.)
 - Motion parallax
 - Kinetic depth effect
 - * Binocular depth perception - Stereopsis

GEOMETRY OF STEREOPSIS

As we learned before, **stereopsis** is considered the most important benefit of binocular vision. Among the different levels of binocular fusion (Worth's degrees of fusion), the highest is stereopsis. Stereopsis provides us with extremely fine depth perception at near and significantly enhances our space perception.

The stereo acuity test is one of the most important tests you can do in a pediatric eye exam, because it provides much information about the development of a child's visual system.

Stereopsis is the unique sense of depth perception that is stimulated by **retinal disparity**.

Q. What is retinal disparity?

A. Retinal disparity is a mismatch in the visual directions for an object seen in the two eyes.

Stereopsis is based on the fact that each eye views the world from a slightly different position. Whereas motion parallax, a monocular depth cue, provides different views of an object at different times, stereopsis takes advantage of the fact that our eyes provide different views of an object at the same time. This is sometimes referred to as **binocular parallax**.

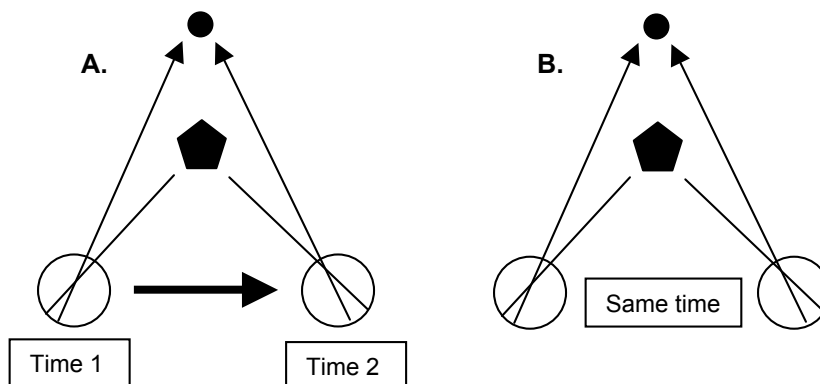


Figure 1. Comparison of motion parallax (A) and binocular parallax (B)

Since stereopsis provides a *relative* (not *absolute*) sense of depth perception, it can only exist when at least two objects are seen. Consider the case of the eyes fixating on a near object (assume no fixation disparity). For any other object to be seen in stereoscopic depth, it must be located either before or behind the fixation point. In the case of objects located in the peripheral field, they must be located distal to or proximal to the horopter to stimulate stereopsis.

Q. Why would there be no sense of stereoscopic depth between the fixation point and another point that is located on the horopter?

A. Any points located on the horopter have the same visual direction in the two eyes; that is, no disparity.

Any object located off the horopter would have a small amount of disparity in the retinal images of the two eyes. That is, the retinal images will have slightly different visual directions (oculocentric directions) in the two eyes. This *disparity on the retina is what stimulates stereopsis*, and the amount of disparity can be computed in object space by the geometry shown in Figure 2.

When two objects are located at different distances, the vergence angle to each is different. The depth interval may be quantified as a linear distance (ΔD), but we usually express disparity as an angular difference. The angular disparity in *object space* is referred to as the **geometric disparity**.

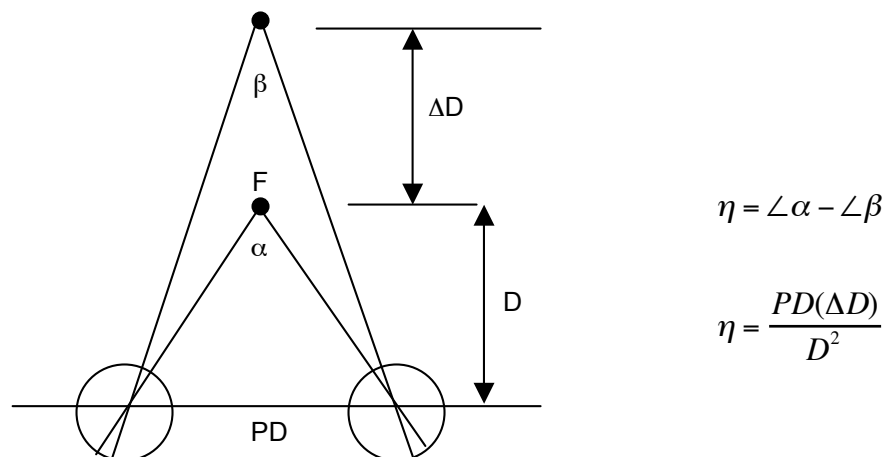


Figure 2. The geometry of stereopsis.

Geometric disparity (η) is the difference between angles α and β , shown in Figure 2, above. You can also estimate the angular disparity using the lower formula, which uses the offset distance (ΔD), the distance to fixation (D) and the PD. If all the linear units are the same (i.e. millimeters, or meters), the value for η will be in radians. The formula for computing binocular disparity (η) from the linear offset (ΔD) is derived as follows:

$$\begin{aligned} \angle\alpha &= \frac{PD}{D} & \angle\beta &= \frac{PD}{(D+\Delta D)} \\ \eta &= \angle\alpha - \angle\beta \\ \therefore \eta &= \frac{PD}{D} - \frac{PD}{(D+\Delta D)} = \frac{PD(D+\Delta D) - PD(D)}{D(D+\Delta D)} \\ \eta &= \frac{PD(D) + PD(\Delta D) - PD(D)}{D(D+\Delta D)} \\ \eta &= \frac{PD(\Delta D)}{D^2 + D(\Delta D)} \end{aligned} \tag{1}$$

When the fixation distance is small, for example within arm's length, the value for ΔD will be very small compared to D , so the second term in the denominator, $D(\Delta D)$, will be very close to zero. It is sometimes dropped to simplify the disparity formula to:

$$\eta = \frac{PD(\Delta D)}{D^2} \tag{2}$$

Note however that for long fixation distances, when ΔD is large, the Equation 2 can lead to significant errors. If Equation 2 gives an unreasonable answer, it is better to use Equation 1, which is simply the difference between angles α and β .

These formulas compute the disparity in radians (if PD , ΔD and D are all in the same units). To convert the value for η to arc seconds, which is the unit normally used for stereoscopic disparity, you must multiply η by 206,265. You will therefore sometimes see this formula written as follows:

$$\eta = \frac{PD(\Delta D)}{D^2} (206,265) \tag{3}$$

Note that, in this case, the value for η will be in arc seconds.

In some binocular vision references, the geometric disparity angle is computed as the difference between angles l and r shown in Figure 3, below. This gives the same answer as the difference between angles α and β , as long as you observe a consistent sign convention when measuring the angles.

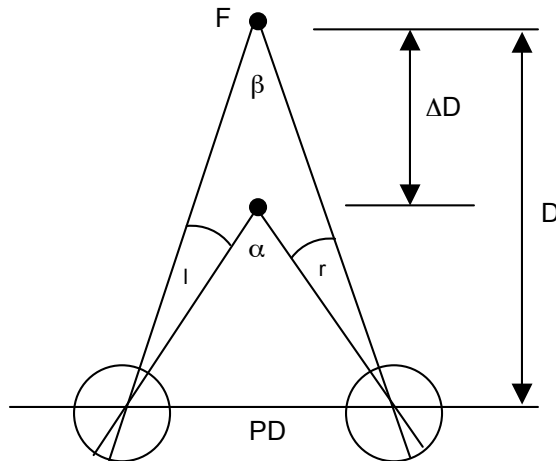


Figure 3. The angles referred to in stereopsis problems.

RELATIONSHIP BETWEEN GEOMETRIC AND RETINAL DISPARITY

Recall that **geometric disparity** is the difference in vergence angles to the fixation point and another object. When considering stereopsis we usually compute the geometric disparity in object space. But, by definition, disparity refers to a difference in visual direction to objects seen by the two eyes. What is the connection between geometric disparity, computed in object space and the visual direction of objects projects onto the retina? Figure 4 demonstrates that retinal disparity is equal to geometric disparity as defined above.

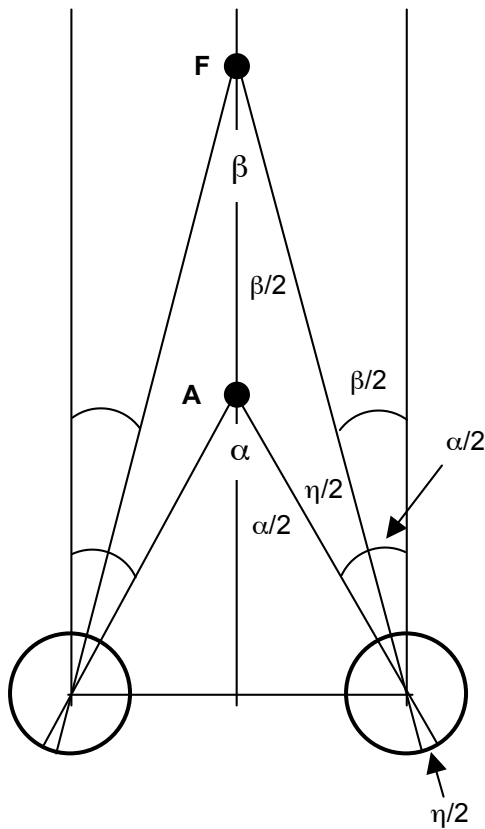


Figure 4. Geometric and retinal disparity.

Consider an object that is foveally fixated (F) and another object located on the midline (A). On previous diagrams, we defined the horizontal geometric disparity (η) as angle α - angle β :

$$\eta = \alpha - \beta. \quad (4)$$

If we divide angles α and β in the midline, we note that the alternate interior angles ($\alpha/2$) and ($\beta/2$), can be measured from the right line, which points straight ahead, parallel to the midline. For the right eye, the angular half-interval between points F and A is:

$$\eta/2 = \alpha/2 - \beta/2 \quad (5)$$

Note that angle $\eta/2$ is the angular size of the interval (FA) on the retina of each eye. On the right retina, the image of point A is located an angle of $\eta/2$ to the right (temporal) of the fovea. The same geometry would apply to the left eye, except that it would be the mirror image of the right. Therefore, on the left retina, the image of A will be located an angle of $\eta/2$ to the left of the fovea. The image of A falls on non-corresponding retinal points, and the total angular disparity is equal to:

$$\eta/2 + \eta/2 = \eta. \quad (7)$$

Therefore, the total retinal disparity for point A (when fixating on point F) is equal to the geometric disparity angle η , which is the difference between angles α and β . To summarize:

- Any object located nearer or farther than the fixation point will have some angle of geometric disparity, relative to the fixation point.
- Since the foveas are directed toward the fixation point, any object located nearer or farther will stimulate non-corresponding retinal points.
- Non-corresponding retinal points are disparate points; that is, they have different visual directions on each retina.
- The total angular retinal disparity (summed between the eyes) will be equal to the geometric disparity.

Geometric disparity, which is caused by objects in space that are located off the horopter, cause retinal disparity. This retinal disparity is the stimulus that leads to stereoscopic depth perception.

BINOCULAR DISPARITY AND DEPTH PERCEPTION

Because of the horizontal separation of the eyes, horizontal geometric disparities give rise to stereoscopic depth perception. Horizontal geometric disparities are a much more powerful depth cue at near.

- Given a fixed linear depth interval (for example, 7 mm) between two points, binocular parallax angles are larger for near objects.
- Therefore, the differences between parallax angles (geometric disparity), are also larger.

This is illustrated by **Fig. 5-5 in Borish's Clinical Refraction**.

- In that figure, the geometric disparity is related to the length of the pencil.
- The disparity angle is smaller when the pencil is farther away.
- If the disparity angle becomes smaller, does the pencil appear to become shorter when it is moved further away?
- If the perceived distances between objects was a direct function of horizontal geometric disparity, then the pencil should appear to become shorter as it is moved further away.

The fact that this does not happen is evidence that the visual system depends on other information besides disparity to compute the relative distances to or sizes of objects. The visual system makes use of all the cues, monocular and binocular, (as well as other information, such as known object size) when computing the perception of distance.

SUMMARY OF FACTS ABOUT STEREOPSIS

- Stereopsis is a special sense of depth perception that is unique to binocular vision.
- The depth perception is *relative*, not absolute.
- It requires at least two objects for comparison.
- It is possible because of **binocular parallax**; i.e., the two eyes view objects from different positions.
- The image of the fixated object falls on corresponding points (foveas), but objects nearer or farther from fixation fall on disparate retinal points.
- **Retinal disparity** is the stimulus for stereopsis.
- Retinal disparity can be measured in object space. This is referred to as **geometric disparity**. As shown in Figure 3, geometric disparity is equal to angle $\alpha - \beta$ or it may also be specified as angle l-r. You can also specify the disparity in linear terms, such as the distance ΔD .

Stereopsis is possible over a range of retinal disparities. The minimum disparity that can be used to develop a sense of stereopsis is **2-10 arc seconds**. It is possible to see images in stereopsis with disparities as large as large as 600 arc seconds (10 arc minutes).

Recall the approximate formula that relates the linear depth (from fixation) to angular disparity.

$$\eta = \frac{PD(\Delta D)}{D^2}$$

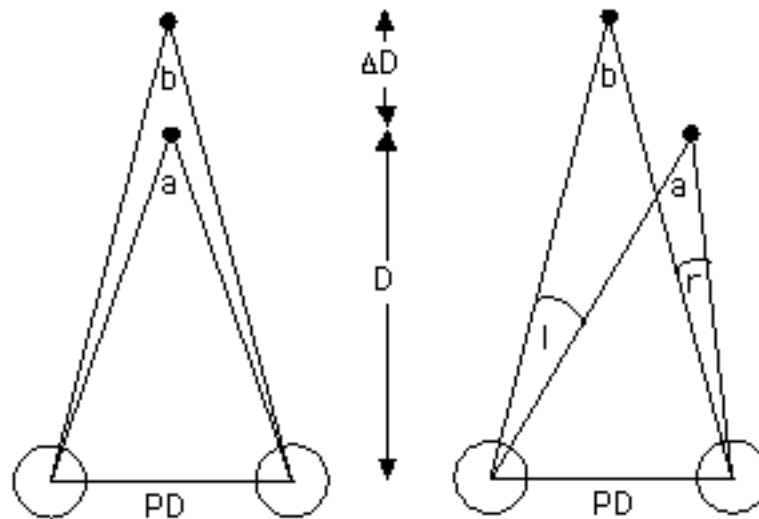


Figure 5. Disparities caused by objects at different distances give rise to retinal disparities and stereopsis.

To get a sense for how good stereopsis is, calculate the following:

Q. What is the angular disparity for an object located 1 mm (ΔD) nearer that the fixation point, if the fixation distance is 40 cm (400 mm) and the PD is 64 mm?

$$\eta = \frac{PD(\Delta D)}{D^2} = \frac{64(1)}{(400)^2} = \frac{64}{160000} = 0.0004$$

This gives the answer in radians. Convert to arc seconds. This is equivalent to 82.5 arc seconds, a large disparity!

Q. In theory, a person with normal binocular vision should be capable of seeing stereopsis with as little as 10 arc seconds of disparity. How much of a distance offset (ΔD) corresponds to this amount of disparity? Use the same parameters for PD and D as the previous problem.

First, convert 10 arc seconds to radians. It is equal to 0.000048 radians. Rearrange the equation and solve for ΔD .

$$\frac{\eta(D^2)}{PD} = \Delta D = \frac{(0.000048)(400^2)}{64} = 0.12 \text{ mm}$$

At a distance of 40 cm, you should be able to tell, using stereopsis, that another object is closer than the fixation point when it has been displaced only 0.12 mm. This shows how amazingly effective binocular stereopsis is for judging relative depth at near distances.

Q. How will differences in PD affect stereopsis? (Refer to the formula.)

A. For the same offset and fixation distance, a larger PD will cause a greater disparity. The disparity is directly proportional to the PD.

LIMITS OF STEREOPSIS

If the amount of disparity is too small, it is insufficient to stimulate stereopsis. As mentioned above, most textbooks consider the angular disparity threshold for stereopsis, also known as the **stereoacuity threshold**, to be **2-10 arc seconds**, which is a very small angle.

If an object is moved further away from the fixation point, its geometric and retinal disparity will increase. Eventually the retinal disparity will become so large that it will exceed Panum's area, and the object will be seen in diplopia. This is close to the maximum amount of disparity that can cause stereopsis.

Stereopsis is possible over a range of distances that extend both distal and proximal to the horopter. This is illustrated in **Fig. 5-6 in Borish's Clinical Refraction**, which shows:

- The fixation point and horopter are located at the center of the zone of stereopsis.
- On either side of the horopter is the zone of binocular fusion, where images are seen in **haploopia** (single vision). This corresponds to the spatial extent of Panum's space (area)
- Beyond this, images are seen in diplopia.
- The left side of the figure shows a narrow zone on either side of the horopter in which **patent stereopsis** is possible. This is the term used by Ogle for high quality stereopsis, in which the perceived magnitude of object depth (quantitative stereopsis) is proportional to the disparity. This zone extends a short distance beyond Panum's area.
- Beyond this is a zone in which objects can be roughly perceived as being either nearer or farther than fixation, but the magnitude of depth cannot be perceived. Since they are beyond Panum's area, they are also seen as diplopia. This is known as the region of **qualitative or latent stereopsis**.
- Beyond this, disparity is too great; objects are simply seen as double, but with no perception of stereoscopic depth.
- Note that the zones are broader in the peripheral visual field.