

The Hall Effect

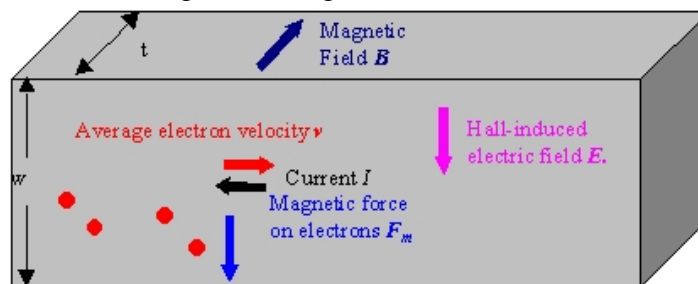
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1 Introduction

If a conductor carrying a current is immersed in a magnetic field, the charge carriers in the conductor may experience an additional force due to this field. The charge carriers are deflected to the side of the conductor as a result of this force, creating a potential difference across the conductor at right angles to the current flow. This process is known as the Hall Effect.

To understand how the Hall effect works, consider the situation shown in the diagram below. In this situation, the electrons are forced downward by the action of the magnetic field. This causes the top of the conductor to become positive with respect to the bottom. This causes a downward directed electric field to develop in the conductor. This field produces an upward force on the electrons. Equilibrium is reached when the electric force is equal to the magnetic force. This condition is

Figure 1: Origins of the Hall Effect



satisfied when

$$eE = evB$$

The induced electric field produces a potential difference across the conductor at a right angle to the direction of current flow. Its magnitude is

$$V_H = Ew = vBw$$

where w is the width of the conductor. This implies that the Hall voltage is proportional to both the electron drift velocity and the applied field. Note also that in this instance that the Hall-induced potential decreases from top to bottom. Suppose instead that the charges carrying the current had been positive. In this case, the charges average drift velocity would be in the same direction as the current; the magnetic force on them would again force them in the downward direction. In this case, the Hall potential then increases from top to bottom. The sign of the Hall voltage can therefore be used to determine the sign of the charge carriers in a conductor.

2 Theory

2.1 Relation Between Current and Drift Velocity

Consider the situation shown in the diagram below. The current passing through a cross-section of the conductor is found by dividing the amount of charge by the time it takes to pass through. The electrons which pass through in time T are those contained within a volume bounded by the cross-section of thickness vT . The total number of electrons in this volume is $nvTwt$, where n is the number density of electrons in the conductor and t is its thickness.

It follows that

$$I = \frac{Q}{T} = \frac{envTwt}{T} = envwt$$

and therefore

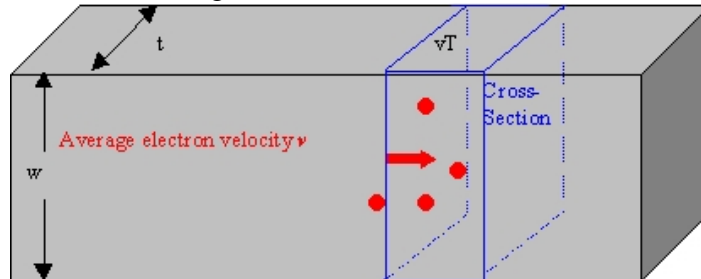
$$v = \frac{I}{enwt}$$

The Hall voltage can then be expressed as

$$V_H = \frac{IB}{ent} \equiv R_H \frac{IB}{t} \tag{1}$$

The quantity $R_H = \frac{1}{en}$ is called the Hall coefficient.

Figure 2: Hall effect on a current



2.2 Electric Current in Semiconductors

2.2.1 Basic Principles

In crystalline solids, the atoms are so closely spaced that their electron wave functions overlap; it then becomes difficult to associate electrons with particular atoms. As a result Pauli exclusion, the discrete energy levels associated with individual atoms become bands of very closely spaced levels; electron energies within the band can then be approximated as a continuous variable. In metals, the outer (or valence) energy band is only partially filled by electrons; these can then easily acquire additional kinetic energy from the application of an electric field. As a result, metals are conductors; that is, they can easily carry an electric current. In nonmetals, the valence energy band is filled. The next higher band is called the conduction band. In insulators, there is a large energy gap between this band and the valence band. In semiconductors, this gap is small. Thermal effects can then push a significant number of electrons into the conduction band, leaving a "hole" or unoccupied energy state in the valence band. Both conduction band electrons and holes can easily change their kinetic energies in response to an electric field and can then serve as carriers for the electric current. The holes behave like positive charges in response to the applied field. Both carriers are known as intrinsic carriers. Their numbers increase as temperature is increased; this results in the resistance of semiconductor decreasing with increasing temperature. Note, however, that the intrinsic carriers cannot produce a Hall voltage, since each is acting in opposite ways.

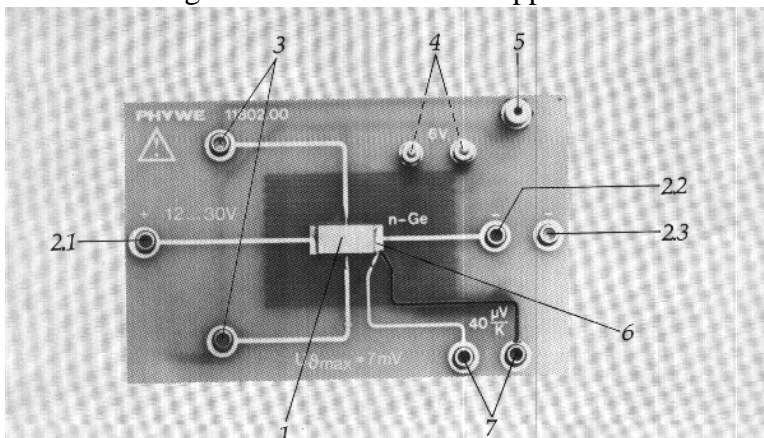
2.2.2 Doped Semiconductors

In a doped semiconductor, the crystal is contaminated by doping with an impurity. If the doping material has one more valence electron than the base material, an n-type semiconductor is produced. As an example, consider the case where germanium is doped with arsenic. The energy levels of the outer electrons in arsenic is slightly different from those in germanium and lie just slightly below the conducting band. As a result, arsenic electrons can be displaced into the conducting band without leaving holes in the valence band. As a result, electrons predominate as the charge carrier in these substances. In a p-type semiconductor, the doping material has one less valence electron than the base material. An example of this occurs when germanium is doped with gallium. The energy levels of the gallium's valence shell are just above the valence band of the germanium and leave holes (unoccupied) states in this energy region. As a result, thermal effects can move some germanium electron's into these holes, creating holes in the valence band without offsetting electrons in the conducting band. In this instance, holes are the majority charge carriers. Charge carriers resulting from doping are known as extrinsic carriers. Over the temperature range in which extrinsic conduction predominates, the resistance of the semiconductor rises with temperature, just as it does in a metal. Once intrinsic conduction becomes important, the resistivity falls with rising temperature, due to the increase in the number of charge carriers. The Hall Voltage is essentially independent of temperature over the extrinsic conduction range (provided that the current is held constant). Once intrinsic conduction becomes important; the Hall Voltage falls as the temperature rises. This is due to the fact that both types of charge carriers are created in equal numbers.

2.3 The Hall Effect Apparatus

The diagram below shows the layout of the Hall effect apparatus. The Hall voltage can be read by connecting across the terminals (3). Drive voltage is applied to the terminals 2.1 and either 2.2 or 2.3. If 2.3 is used, a current limiter is engaged which maintains the current through the crystal at about 30 mA. If 2.2 is used, this function is bypassed; the user then must take care that the current does not greatly exceed this value. The knob (5) is used to adjust for the offset voltage across the Hall voltage terminals (3). Unless the taps for these are directly opposite one another (a manufacturing impossibility) there will be a potential difference induced between these when current is flowing even with no applied magnetic field as a result of the resistance of the crystal. The plugs (4) attach the board to

Figure 3: The Hall Effect Apparatus



its support; they can also be used to apply current to a heater at the base of the crystal. The jacks (4) connect to a thermocouple which can be used to measure the temperature of the crystal.

Caution #1: Germanium crystals are brittle and are therefore fragile. You must take great care not to bend or twist the plate support during assembly and disassembly.

To affix the plate to the support rod, the two plugs (4) are pushed into the sockets by pressing on the front of the plates between the nuts that secure them. To remove the plate, the back of the plate near these plugs is pressed with two fingers and pulled away from the rod. When making connections to individual sockets, support the plate by pressing your hand against the back of the plate.

Caution #2: The maximum allowable current in the crystal is 50 mA. The current limiter is set to 30 mA, which yields a generous safety margin. However, some measurements will be made with the limiter bypassed. When operating in this mode, you must take care not to exceed this limit.

3 Experimental Procedure

Eq. (1) predicts that the Hall voltage is directly proportional to either the applied field or the crystal current (provided that the other parameter is held constant).

We will use a variable voltage supply and permanent magnets to test these notions. The permanent magnets are attached to opposite sides of a C-clamp (see the picture below). The magnetic field strength is varied by adjusting the gap between the magnets.

3.1 Variation of the Magnetic Field Strength with Gap Width.

First, you will determine the range for which the magnetic field varies linearly with gap width. This is done by the following procedure.

1. Connect the crystal board to the support rod using the plugs (4).
2. Adjust the C-clamp so that the magnets are about 4 cm apart. Determine the pitch of the adjusting screw by measuring the linear advance of the screw for some number of turns.
3. Connect a voltmeter across the terminals (3). Connect the power supply to the terminals (2.1) and (2.3). Turn on the power supply and set the voltage to around 12 V. Any voltage that appears across (3) is due to offset of the connections to the crystal and should be eliminated by adjusting (5).
4. Place the magnet assembly on the jack so that the germanium crystal is centered between the magnet poles.
5. Make a series of measurements of the Hall Voltage vs. magnet position. Rotate the screw by one revolution between each measurement. The magnet gap can be determined without removing the magnets by subtracting the total number of turns times the pitch from the initial gap width. Make as many measurements as you can, however, be careful not to let the magnets touch the face of the crystal. If this occurs, the force of the jaws may damage the crystal.
6. Plot a graph of Hall Voltage vs. magnet gap. According to Eq. (1), this voltage is directly proportional to the magnetic field strength (with constant input current). This graph can therefore be used to determine the range over which magnetic field varies linearly with gap width.

3.2 Hall Voltage vs. Current Measurements

1. Connect an ammeter and the 175 W rheostat in series with the crystal, and connect the combination to a variable power supply. One lead of the power supply should be plugged into (2.1); the ammeter should connect at (2.2) (this bypasses the current limiter). Slowly increase the voltage until you get a current of about 10 mA. Use the knob (5) to cancel any offset voltage.
2. Set the magnet gap to a setting about 1/3 of the way in from the maximum gap in the linear range. Reposition the magnet with the crystal between the poles. Measure the Hall Voltage and adjust the screw so that the gap is at its minimum safe setting. Measure the Hall voltage in this position.
3. Increase the power supply voltage until the current reaches 15 mA and repeat the measurements in step 2. Continue this process for currents up to 35 mA. Remember that each time the current is increased, the offset voltage must be rechecked.

3.3 Temperature Measurements

1. Remove the ammeter and the rheostat from the circuit and reconnect the power supply across (2.1) and (2.3). Connect the other rheostat across one of the 5 V taps on the power supply; then connect its variable output and ground to the plugs (4). Set the rheostat to the zero voltage position.
2. Connect the CBL thermocouple amplifier to the jacks (7). Plug the amplifier output into Channel 1. Switch the CBL into voltage mode. Move the rheostat to about the 1/5 of the maximum setting. Wait a minute to see if the temperature changes. If it does, wait until it stabilizes, then record the Hall and thermocouple voltages.
3. Increase the rheostat setting and repeat the measurement procedure. Try to take a total of ten measurements as you procedure to the maximum setting. Once you have identified the point at which the Hall voltage begins to change, take additional readings around this point. Use the smallest steps that produce a detectable change in the temperature of the crystal.

4 Data Analysis

1. For each magnetic field strength in part (2), plot a graph of Hall Voltage vs. Current (you may plot them on the same graph). Is your result consistent with Eq. (1)?
2. Plot a graph of Hall Voltage vs. Temperature. Determine from your graph the temperature at which the transition from extrinsic to intrinsic conduction occurs.