Wave Optics

2nd November 2001

1 Theory

1.1 Diffraction

Figure 1 shows diffraction of a wave through a narrow aperture. The degree of spreading in the outgoing wavefronts is large when the aperture size is comparable to or smaller than the wavelength, and is small when the opening is much bigger than the wavelength. The shape of the wavefronts may be derived using Huygens principle; each point in a wavefront is treated as the source of spherical wave-lets whose superposition generates the next wavefront.

1.2 Single Slit Diffraction

Consider the situation shown in Figure 2. The intensity pattern seen on the projection screen is the sum of the contributions from each point in the wavefront in the slit. These contributions will differ in phase at the screen because the wave-lets travel different distances to reach a common point on the screen. Consider first a point in the slit a distance x from the midpoint of the slit. Assuming that the distance D to the projection screen is much greater than the slit width a, the extra distance is approximately equal to

$$\Delta L \approx x \sin \alpha$$

Let A_0 equal the amplitude of wavefront in the slit. If this is constant over the entire width of the slit, then a small section of width dx will contribute an amount $\frac{A_0 dx}{a}$ to the total. The wave-let originating a x will be out of phase with the one

Figure 1: Diffraction through a narrow aperture

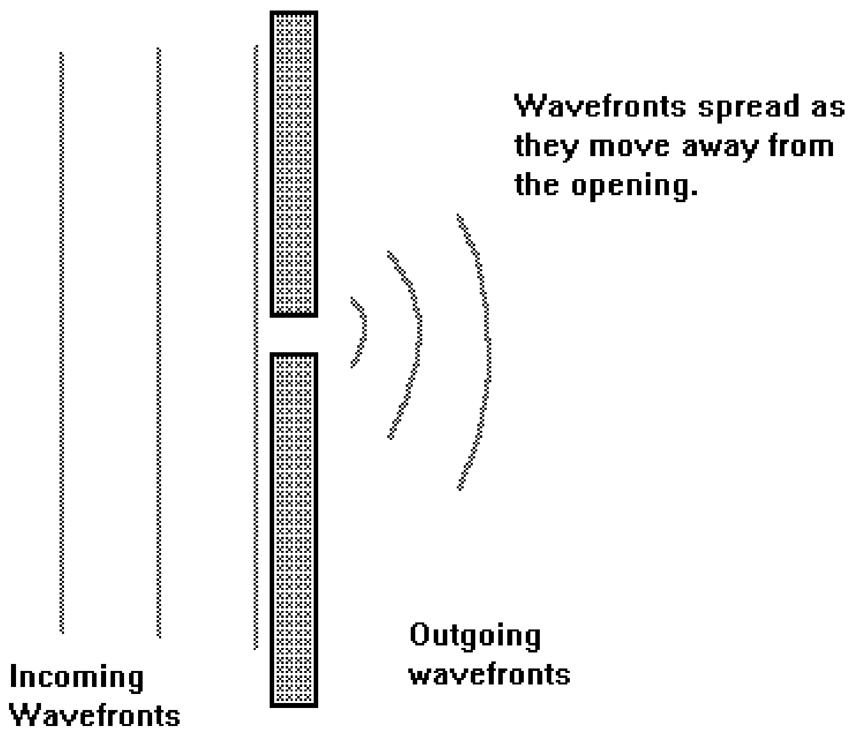
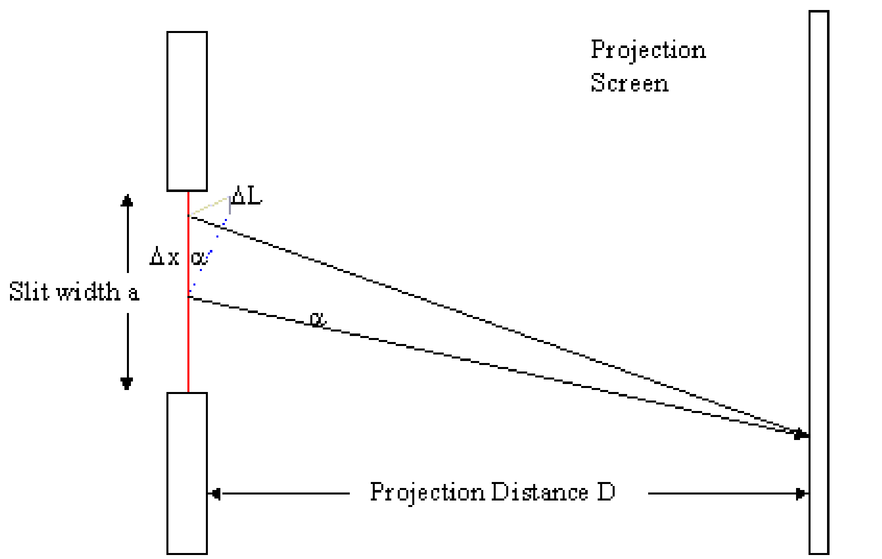


Figure 2: Path difference ΔL for a single slit



from the midpoint by a phase angle $\phi = \frac{2\pi\Delta L}{\lambda}$. Hence this wave-let's contribution is

$$\frac{A_0 dx \cos\left(\frac{2\pi x \sin\alpha}{\lambda}\right)}{a}$$

The amplitude of the wave at any point on the projection screen is then found by integrating this expression over the whole slit. If we define a new variable

$$\theta = \frac{\pi a \sin\alpha}{\lambda}$$

this expression will simplify to

$$A = A_0 \frac{\sin\theta}{\theta} \tag{1}$$

The intensity of the light is proportional to the square of the amplitude. A plot of the intensity pattern is shown in Figure . Note that the *y*-axis scaling is logarithmic. The zeros in the intensity pattern occur at the locations for which $\sin\theta = 0$ (other than $\theta = 0$). This in turn requires that

$$\frac{\pi a \sin\alpha}{\lambda} = m\pi \quad (m = 1, 2, 3 \ldots)$$

and hence

$$a \sin\alpha = m\lambda \tag{2}$$

Since the intensity is proportional to the square of the amplitude, Eq. 1 tells us that

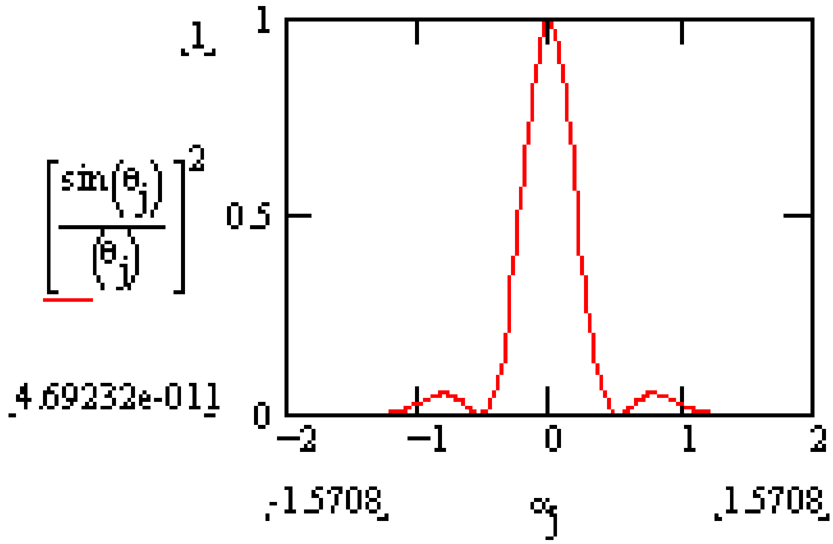
$$I = \frac{I_0 \sin^2\theta}{\theta^2} \tag{3}$$

1.3 Diffraction by a Double-slit

The double-slit diffraction pattern may be thought of as the result of two single-slit patterns that are superimposed so that they nearly coincide. To compute the sum, note that the light from one slit lags the other in phase as a result of the difference in path lengths between them (see Figure 4). The extra distance is $\Delta L = d \sin\alpha$ where *d* is the slit separation. The phase angle between the two is

$$\phi = \frac{2\pi d \sin\alpha}{\lambda}$$

Figure 3: Intensity vs. angle plot for a single slit



The overall amplitude is found by multiplying the amplitude of the lagging contributor by $\cos\phi$ and adding it to the first

$$A = \frac{A_0 \sin\theta}{\theta}\left(1 + \cos\left(\frac{2\pi d \sin\alpha}{\lambda}\right)\right)$$

The condition for a maximum is

$$\frac{2\pi d \sin\alpha}{\lambda} = 2n\pi \quad (n = 1, 2, 3 \ldots) \tag{4}$$

while the condition for a minimum is

$$\frac{2\pi d \sin\alpha}{\lambda} = (2n+1)\pi \quad (n = 1, 2, 3 \ldots) \tag{5}$$

The intensity is again proportional to the square of the amplitude.

$$I = I_0 \left(\frac{\sin\theta}{\theta}\right)^2 \left(1 + \cos\left(\frac{2\pi d \sin\alpha}{\lambda}\right)\right)^2 \tag{6}$$

The typical form of a double-slit intensity pattern is shown in Figure 4.

Figure 4: Path difference in terms of the angle α

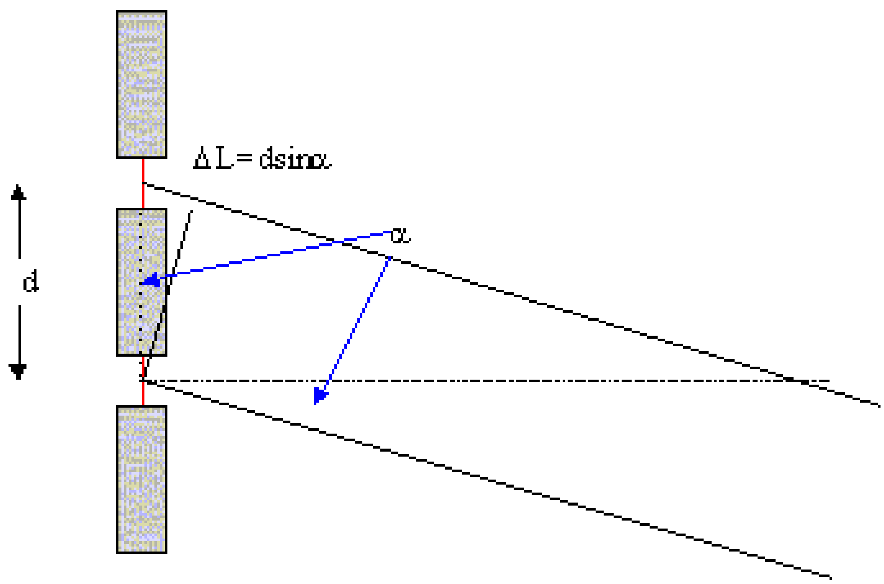


Figure 5: Intensity graph for a double slit

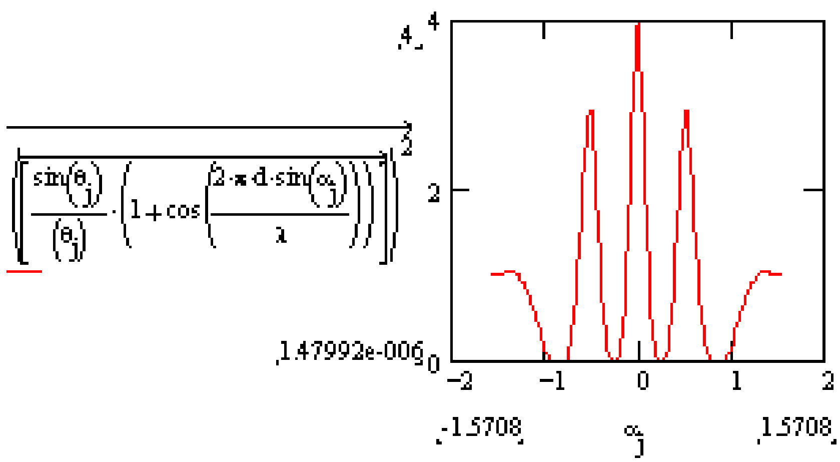
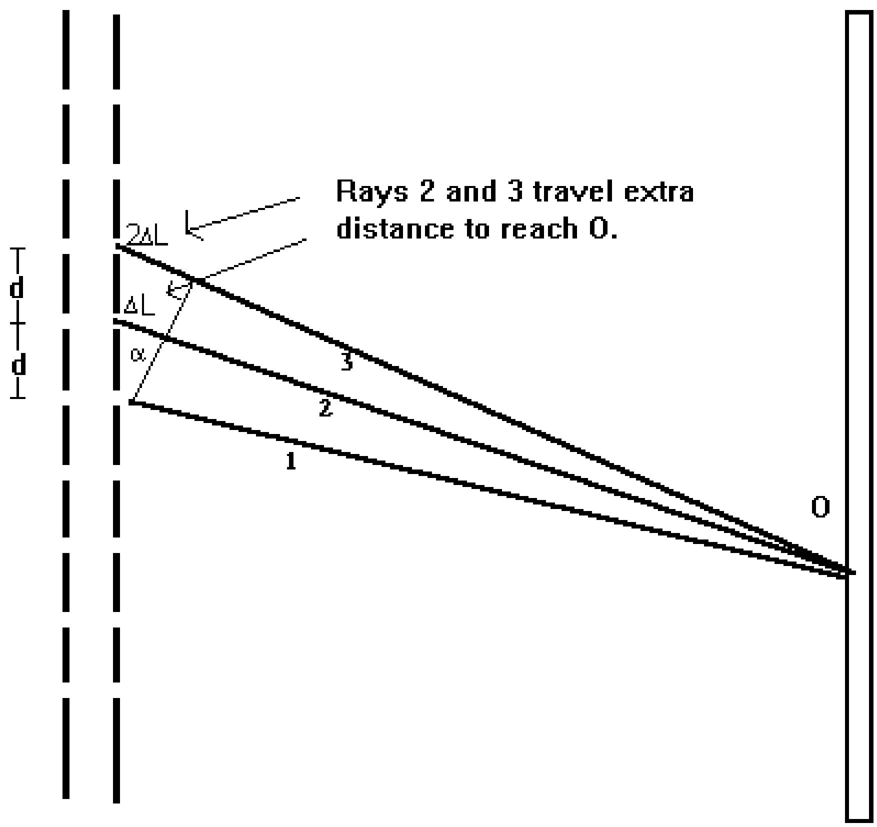


Figure 6: Path lengths for adjacent slits in a grating



1.4 Diffraction Grating

The diffraction grating pattern can be understood using the above results for the double slit. A typical grating has hundreds of small openings through which light is passed. Diffraction causes light from different openings to overlap in the region behind the grating. Since light from different openings travels different distances, the wave-lets will have different relative phases when they arrive at a particular location. The path lengths for each are related; if adjacent slits have a difference ΔL, the next one will differ from the first by $2\Delta L$, and so on (see Figure 6). If the adjacent slits are separated by a, the intensity at a particular point will have the form

$$A = \frac{A_0 \sin\theta}{N\theta} \sum_{n=0}^{N} \cos\left(\frac{2n\pi d \sin\alpha}{\lambda}\right)$$

The sum in this expression will in most cases essentially averages the cosine function over several arguments. For most viewing angles α, this will result in virtually complete cancellation, so the region appears dark. If, however, α satisfies the condition that the distances traveled by light from adjacent slits differ by a whole number of wavelength, constructive interference will occur. The $\alpha's$ that satisfy this condition are the same as those that produce constructive interference for the double slit.

2 Experimental Procedure

2.1 Introduction

Data collection for this experiment will be performed using the DataStudio software package. The program receives data from the Science Workshop Interface. This unit monitors the sensors for light and angular motion and reports their values, along with the collection time. By default, the DataStudio software reports the values as a graph of sensor reading vs time; however, it can also plot angular position vs light intensity, which is the form we will use in this experiment. For a detailed information on how to use DataStudio, refer to its on-line help.

2.2 Double-slit

In this part, you will use the interference pattern of double-slits to determine the wavelength of a diode laser. You will also plot graphs of intensity vs. angular position and compare them with the predictions of Eq. 6.

1. Begin by selecting one of the double slits on the Pasco Multiple Slit Accessory. Align the equipment so that a clear diffraction pattern is projected. Be sure that the Linear Motion Translator is positioned perpendicular to the axis of the optical bench.

2. Position the detector unit at one stop and start DataStudio. Select the “Create Experiment” option. Three windows will appear, labeled “Data”, “Display”, and “Experiment Setup”. Connect the rotary motion sensor and light

probe to the Science Workshop Interface in the Experiment Setup window.[1] Double-click on the Rotary Motion Sensor and select the Position Measurement option. Be sure that the calibration option is set to "Rack" Start data collection and slowly sweep the light probe over to the other stop. Now terminate data collection. Your data files will appear in the Data window. One will contain the position of the light probe vs. time, the other will contain the intensity reads vs. time. Display the intensity vs. time data by dragging that data set to the "Graph" option in the Display window. To change the graph to Intensity vs. position, drag your position data to the x-axis of this graph. Be sure that enough points are included to accurately represent the intensity pattern. If not, discard your data sets and repeat the process with a slower sweep rate or with an increased sampling rate.

3. Once you have obtained a good data set, export it as a text file. You can then import it into Mathcad for further processing. First, determine the positions of each of the constructive and destructive interference points in the pattern. Calculate the value of the angle α for each using the relation

$$\tan\alpha = \frac{y}{D}$$

   where y is the distance of a point from the center of the diffraction pattern (central diffraction peak), and D is the projection distance given in Figure 2. Use Eqs. 4 and 5 to calculate the wavelength of the laser from this data and the slit separation d. Average your results for the wavelength and determine the standard error.

4. Select a different double-slit and repeat steps 1 through 3. Be sure to use a different slit width a as well as a different separation d.

5. Finally, check the agreement between your data and the intensity pattern predicted by Eq. 6. To do this, graph the data set and the function in same plot.[2] Scale the function graph so that I_0 is equal to the highest intensity value recorded in your data set.

[1] This is a "drag and drop" operation. Consult the on-line help for DataStudio for additional details.

[2] For clarity, create separate plots for each pair of slits.

2.3 Single-slit

1. Remove the Double Slit Accessory from the bench and replace it with the variable single-slit apparatus. Set the slit width so that a clear diffraction pattern is obtained.[3] Use DataStudio to obtain a set of intensity measurements vs. position in the same fashion as you did for the double-slit. Save the data as a text file and import it into Mathcad.

2. Calculate a value for the slit width using Eq. 2. Average your results to obtain a best value.

3. Using the data obtained in the previous step, construct a graph that compares the actual intensity pattern to the predictions of Eq. 3.

4. Repeat for second slit width.

2.4 Questions

1. For the double slit, how does the distance between maxima change as the slit separation decreases?

2. Is the width of the diffraction pattern for a double-slit affected by the slit width? If so, how does it change?

3. For the single slit, how does the distance between minima change as the slit width increases?

[3]Both the central peak and at least one pair of side peaks should be clearly visible.